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$$\begin{aligned} \therefore p &= 1 - \frac{4}{5\pi r^3} \left[ \int_{\frac{1}{2}r}^r \int_0^\theta [15(\frac{1}{2}\pi - \theta)\cos\theta - 10\cos\theta + 10\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta - \frac{5}{2}\tan\frac{1}{2}\theta\sec^2\frac{1}{2}\theta] \times \right. \\ &\quad \left. x^2 dx d\theta + \int_0^{\frac{1}{2}r} \int_0^{\frac{1}{2}\pi} [15(\frac{1}{2}\pi - \theta)\cos\theta - 10\cos\theta + 10\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta - \frac{5}{2}\tan\frac{1}{2}\theta\sec^2\frac{1}{2}\theta] x^2 dx d\theta \right] \\ &= 1 - \frac{4}{5\pi r^3} \left[ \int_{\frac{1}{2}r}^r \left( 15(r-x)\cos^{-1}\left(\frac{r-x}{x}\right) - 10r - 20\sqrt{2rx-r^2} \right) \right. \\ &\quad \left. + \frac{5}{2}x - \frac{5x^2}{x + \sqrt{2rx-r^2}} \right) dx + \frac{1}{2} \int_0^{\frac{1}{2}r} x^2 dx \Big] = \frac{4}{\pi} (8\log 2 - 5). \end{aligned}$$

Also solved with same result by the *PROPOSER*.

### MISCELLANEOUS.

107. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

The index of refraction of a medium varying inversely as the square root of the distance, prove that the path of a ray of light in the medium is a cycloid.

Solution by the *PROPOSER*.

Taking the axis of  $y$  in the given plane and that of  $x$  at right angles to the  $y$  axis, letting  $\mu = k/\sqrt{x}$  be the index of refraction, and  $p = dy/dx$ , we have, by the usual theory, for the differential equation to the path

$$\frac{dp/dx}{1+p^2} = \frac{1}{\mu} \left[ \frac{d\mu}{dy} - \frac{d\mu}{dx} \frac{dy}{dx} \right] \dots (1). \quad \text{We have } \frac{d\mu}{dx} = -\frac{k}{2x^{\frac{3}{2}}} \dots (2),$$

$$\text{and (1) becomes } \frac{dp/dx}{1+p^2} = \frac{p}{2x}, \text{ or } \frac{dp}{p(1+p^2)} = \frac{dx}{2x} \dots (3).$$

$$\text{Integrating, } \log \frac{p}{\sqrt{1+p^2}} = \log \sqrt{x} + C \dots (4).$$

$$\text{Let } p=b, \text{ when } x=a; \text{ then } C = \log \frac{b}{a\sqrt{1+b^2}},$$

and (4) becomes  $p = \frac{dy}{dx} = \frac{ax}{\sqrt{[(a^2/b^2)(1+b^2)x - x^2]}} \dots (5)$ , the differential equation to a cycloid.

Also solved by G. B. M. ZERR, and L. C. WALKER.

108. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

To divide the arc of a cardioid into eight equal parts.

Solution by L. C. WALKER, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Equation of the cordiod is  $r=a(1+\cos\theta)$ , and its complete perimeter is

$$S=2\int_0^\pi [a^2(1+\cos\theta)^2+a^2\sin^2\theta]d\theta=4a\int_0^\pi \cos\frac{1}{2}\theta d\theta=8a,$$

the arc  $S$  being measured from the point where the curve crosses the initial line.

Let  $\theta'$  be the point of the first division; then

$$2a\int_0^{\theta'} \cos\frac{1}{2}\theta d\theta=4a\sin\frac{1}{2}\theta'=a;$$

from which  $\theta_1, \theta_7=\pm 2\sin^{-1}\frac{1}{4}$ . Similarly,  $\theta_2, \theta_6=\pm 2\sin^{-1}\frac{1}{2}$ ,  $\theta_3, \theta_5=\pm 2\sin^{-1}\frac{3}{4}$ ,  $\theta_4=\pi$ , and  $\theta_8=0$ .

Excellent solutions were received from G. B. M. ZERR, and F. P. MATZ.

109. Proposed by J. SCHEFFER, A. M., Hagerstown. Md.

Find the latitude of the place where the sun's center remains above the horizon for a hundred successive days.

Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy. Defiance College, Defiance, O.

I. The solar phenomenon in question will begin May 2 and end August 10. For May 2 at Washington mean noon, according to the Director of United States Nautical Almanac Office, the declination of the sun is  $\delta=+15^\circ 18' 54.7''$ . Representing the hour-angle of the sun by  $h$ , we have  $\cosh=-\tan\delta\tan\phi\dots(1)$ .

Since there is to be no setting of the sun, we may put  $\cosh=-1$ .

$\therefore \lambda=\tan^{-1}\left(\frac{1}{\tan\delta}\right)=\tan^{-1}(\cot\delta)=90^\circ-\delta=74^\circ 41' 5.3''$ , which is the terrestrial latitude of the place required. This solar phenomenon may be observed on Melville Island.

II. The celestial longitude of the sun for mean noon Washington can be determined in various ways to be about  $\psi=41^\circ 34' 25''$ ; the obliquity of the ecliptic may be taken  $\omega=23^\circ 27' 8''$ ; then  $\lambda=90-\delta=90-\sin^{-1}(\sin\psi\times\sin\omega),=74^\circ 43' 16''$ .

Also solved by G. B. M. ZERR, and H. C. WHITAKER.

110. Proposed by E. W. MORELL, South Trowbridge, Vt.

If  $a$  and  $b$  be the sides of a triangle,  $A$  and  $B$  the angles opposite, then will  $\log b - \log a = \cos 3A - \cos 2B + \frac{1}{2}(\cos 4A - \cos 4B) + \frac{1}{8}(\cos 6A - \cos 6B) + \dots$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

$$1 + \cos 2A + \frac{1}{2}\cos 4A + \frac{1}{8}\cos 6A + \dots = -\frac{1}{2}\log(2 - 2\cos 2A) = -\log(2\sin A).$$

$$1 + \cos 2B + \frac{1}{2}\cos 4B + \frac{1}{8}\cos 6B + \dots = -\log(2\sin B).$$

See Trigonometry for the summation of these series.

$$\therefore \cos 2A - \cos 2B + \frac{1}{2}(\cos 4A - \cos 4B) + \dots = \log(\sin B/\sin A) = \log(b/a).$$

Also solved by F. P. MATZ, and J. SCHEFFER.